Chapter - 23 Basic Algebra

23. Basics of Algebra:

23.1 Instead of numbers, algebra uses symbols. The symbols used are usually the letters of the English alphabet.

Thus a, b, c, d ….. p, q …….. x, y z
Usually the small letters (lower case) are used.

Go back to the chapter on substitution where the same ideas are introduced. Come back and read more.

23.2 Symbols are very useful for generalizations.

E.g. What is your name?
My name is Lata.

This is quite OK. But when all the girls in a class repeat this, it generates laughter. When all the boys also say the same, it becomes a joke.

23.2.1 If the teacher insists that all the students should write the above answer, it becomes a ridiculous joke on the education system. What will you do?

What is your name? My name is | * |
* Write your name here.

What is your mother tongue? My mother tongue is | ** |
** Write here Kannada / Hindi / Tamil etc.

(Teachers, use such examples to show that blank spaces can be filled by symbols and those symbols could be explained).

23.2.2 What is your name? My name is x

What is your father's name? My father's name is y

(x, y – Fill up with suitable names. Here x, y are also symbols).

These are examples of filling up the blanks as in English or History exams or in forms for jobs, ration cards etc., Algebra is not very different.

23.3 Instead of word or data substitution as seen above, there can be substitution of x, y etc by numbers. It will then look like maths. Now consider:

A friend gives you US $ 100. How much money do you have?

Ans: I don't know (is OK).
A clever fellow asks, " How much is a dollar in rupees"? "
"I don't know is also OK.
The best answer is - Money in my hand = 100 X x rupees.
(Where x = value of one dollar in rupees on that date)
(Students can make examples like this one)

In the above example use of x, y or any letter helps. To use this effectively, students should be good at substitution.

23.4 The advantage of algebra (i.e., using x or y etc) is that the value of x or y need not be the same. It can change as per the situation in question.

23.4.1 In the above dollar example, let us assume that:
Exchange rate of US dollar (on one date) = Rs. 45

Then US dollars 100 = 100 X 45
= Rs 4500

If the exchange rate today is Rs. 50
Today's worth of US $ 100 = 100 X 50 = 5000 rupees

Both came from the equation, value of 100 US dollars = 100 x x
Where x = rupee value of 1$ (on that day)

Without doing anything you gained Rs. 500

23.5 Go back to substitution chapter & do the same again. Here are some more.

1. $x = 10$, $10x = ?$ Ans: 10 x 10 = 100
2. $x = 10$, $10 + x = ?$ Ans: 10 + 10 = 20
3. $x = 10$, $10 - x = ?$ Ans: 10 – 10 = 0
4. $x = 10$, $x - 9 = ?$ Ans: 10 – 9 = 1
5. $x = 10$, $\frac{10}{x} = ?$ Ans: $\frac{10}{10} = 1$
6. $x = 10$, $\frac{x}{10} = ?$ Ans: $\frac{10}{10} = 1$

Exercises:

a. 10a =? , If a = 8.5 b. 10 + a =? , If a = 90 c. 10 – b =? , If b = 8

d. c – 99 =? , If c = 100 e. $\frac{77}{d}$ = ? , If d = 7 f. $\frac{x}{15}$ = ? , If x =30

[(g) to (o) to be generated by students. One per student OK].

23.6 **More problems**

1. $x = 5$, $\frac{x+5}{5} = ?$ Ans: $\frac{5+5}{5} = \frac{10}{5} = 2$
2. $x = 5$, $\frac{x-5}{5} = ?$ Ans: $\frac{5-5}{5} = \frac{0}{5} = 0$
3. $x = 5$, $\frac{10}{x+5} = ?$ Ans: $\frac{10}{5+5} = \frac{10}{10} = 1$
4. $x = 5$, $\frac{10}{x-3} = ?$ Ans: $\frac{10}{5-3} = \frac{10}{2} = 5$
3. $x = 5$, $\frac{2}{x-3} = ?$ Ans: $\frac{2}{5-3} = \frac{2}{2} = 1$
4. $x = 5$, $\frac{2}{7-x} = ?$ Ans: $\frac{2}{7-5} = \frac{2}{2} = 1$

(7) to (10) to be generated by students.

Exercises: Given a = 11, find

a. $\frac{a+9}{5}$ b. $\frac{a-9}{2}$ c. $\frac{30}{a+4}$ d. $\frac{28}{a-4}$ e. $\frac{3}{a-8}$ f. $\frac{3}{14-a}$

g. $\frac{a-8}{14-a}$ h. $\frac{8-a}{a-14}$

23.7 Substitution of 2 variables

If $x=1, \ y=2$

a) $x+y=?$ Ans: 1 + 2 = 3

b) $y-x=?$ Ans: 2 – 1 = 1

Then ask c) $x-y=?$ (Explain negative numbers) Ans: 1 – 2 = -1

d) $\frac{x}{y} = ?$ Ans: $\frac{1}{2}=\frac{1}{2}$

e) $\frac{y}{x} = ?$ Ans: $\frac{2}{1}=2$

23.7.1 Exercise: Given that x = 20 y = 10 find:

a. $x + y$ b. $x - y$ c. $y - x$ d. $\frac{x}{y}$ e. $\frac{y}{x}$ f. $\frac{x+y}{x-y}$

23.7.2 *If* $x=1, \ y=2$

a) $2x+y=?$ Ans: (2 x 1) + 2 = 2 + 2 = 4

b) $2x-y=?$ Ans: (2 x 1) – 2 = 2 –2 = 0

c) $2y-4x=?$ Ans: (2 x 2) – (4 x 1) = 4 – 4 = 0

d) $\frac{2x}{y}=?$ Ans: $\frac{2 \text{ x } 1}{2}=\frac{2}{2}=1$

e) $\frac{y}{4x}=?$ Ans: $\frac{2}{4 \text{ x } 1}=\frac{2}{4}=\frac{1}{2}$

(Ask students to give x & y different valves and do the above problems in groups. Show the results).

23.7.2 Exercise: Given that x =20, y =10 find:

a. $2x + y$ b. $2x - y$ c. $2y - 4x$ d. $\frac{2x}{y}$ e. $\frac{y}{4x}$

f. $\frac{2x+y}{2x-y}$ g. $\frac{2x+y}{x+2y}$

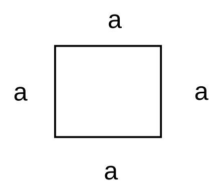
23.8 Teachers, drill this.
You can use number bricks and letter bricks for this.
Let a group of 3 to 5 persons play. One is leader with calculators. Others work out by hand.
They check the results.

23.9 Substitution principle used in some formulas.

23.9.1 Single Variable:
a. A = C + 273 where A = ^{o}K, C = ^{o}C } Temperatures

What is the absolute temperature in ^oK, for 0^o C?
Ans: A = 0 + 273
= 273^oK
What is the room temperature in absolute scale, if the thermometer reads 27^o C.
Ans: A = 27 + 273 = 300^oK

b. The sides of a square are a units. Then Area, A = a^2 square units



What is the area of a plot of 12 meter by 12 meter square?
Area, A = $(12)^2$ = 12 x 12 = 144 square meters = 144 Sq. m

Exercise:
c. Akka is 5 years elder to Thangi. Write a small equation for this. If Akka is 15 years old now, what is Thangi's age? What will be Akka's age when Thangi's age becomes 60?

23.9.2 Two Variables:

A. Distance traveled = Speed x time
Or d = v x t [v for velocity ≈ speed]

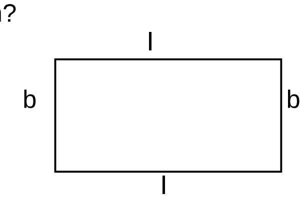
A bus going at an average speed of 50 km/hr. takes 3 hrs to go from Mysore to Bangalore. What is the distance between these two cities?

Ans: d= v x t v= 50 t = 3
= 50 x 3 = 150 km

Exercise:
A1. A cyclist is going at 15 km/hr. How long will he take?

A2. If a taxi took only 2 hours, how fast was the taxi?

B. If l is the length and b is the breadth (=width) of a rectangle, its Area
A = l x b. What is the area of a plot of 10 m x 14 m?
A = l x b l =14 m b = 10m
= 14 x 10
= 140 Sq. m



C. What is the least count (LC) of a Vernier? LC = (MSD) – (VSD) when
MSD = 1 mm, VSD = 0.9 mm.

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Chapter - 24 Basic Operations in Algebra - A

24. Basic Operations in Algebra:
+, -, X, ÷ are the basic operations. We have learnt how to do these on real numbers. Now we will do the same with symbolic numbers (i.e. algebra).

24.1 Addition:
Same symbols will add just like numbers
1 + 1 + 1+ 1 = 4
a + a + a + a = 4a

$x + x + x + x = 4x$
u + u + u+ ………. 30 times = 30u

Since many times addition = multiplication
4a, $4x$, 30u above also mean: 4 X a, 4 X x, 30 X u
i.e. multiplying symbol (X) need not be written.

Problem with numbers: The same cannot be done with numbers.

4 = 41 (this will become forty one i.e. 40 + 1)

Therefore we should write 4 = 4 x 1. This can be written as 4 = 4(1)

Exercises: Say True or False or Yes/No Desirable/Not Desirable

1. b + b + b = 3b
2. c + c + c = c3
3. d + d + d = ddd
4. If a = 1; b = 2, ab = 12
5. If a = 1; b = 2, ab = 2
6. ab = a + b
7. ab = a x b
8. ab = a .b
9. a.b = a x b
10. x^5 is not written; it is written as $5x$
11. x^5 means x x x x x x x x x

24.2 Now try 3 + 3+ 3 = 9 also 3(3)
If we write a + a + a = 3 (a) = 3a
If we write 3a + 3a + 3a = 9(a) = 9a
3 + 2 + 1 = 6
3a + 2a + 1a = 6a
(1a is usually written as a)
Therefore 3a + 2a + a = 6a

Instead of ‘a’ anything can be written. This is the reason this is called algebra.

Thus 3b + 2b + b = 6b
$3x + 2x + x = 6x$
3p + 2p + P = 6p

Exercise:
Students can now go back to exercises of 24.1. They can check whether they have done right or not.

24.3 Mixing of Numbers and Letters:
Numbers all belong to one group. So they can all add up.

Thus 1 + 2 + 3 = some value = A

11 + 12 + 13 = some value = B

(any big number) + (any other number) = Some value = C

Similarly ALL items belonging to ONE GROUP (i.e. algebra) can add up.

i.e. $x + 2x + 3x$ = Some value

$11x + 12x + 13x$ = Some value = Bx

(Any big no.) x + (Any other no.) x = Some value = Cx

24.4 Note for Teachers [Self – learning students can skip this section]

In writing numbers, we are using what is called PLACE VALUE and use digital system (base 10). Thus (4, 42, 421) here value of 4 is different depending upon where it occurs. (Teachers! If you feel like you can digress upon this system. Go if you like to the greatness of zero – how the zero after 1 to 9 is used back again to give 10. Use an abacus if you like. This is fully optional because such a discussion can only be suggested by a manual maker).

24.5 In number system 1111 means 1000 + 100 + 10 + 1
Similarly 5555 = 5000 + 500 + 50 + 5
But aaaa $\neq$ 1000a + 100a + 10a +a
LHS above means a X a X a X a = a^4
RHS above means (1000 + 100 + 10 + 1) a = 1111a
Similarly 5a5a5a5a does not exist. Do not write this way.
Instead 5555a is OK. But a5555 is not written.
This means 5000a + 500a + 50a + 5a

Exercise: Answer right or wrong (√ or X)

a. 1234 = 1000 + 200 + 30 + 4
b. 1234a = 1000 + 200 + 30 + 4 + a
c. 1234a = 1000 + 200 + 30 + 4a
d. 1234a = 1230 + 4a
e. 1234a = 1000a + 200a + 30a + 4a
f. 9876b = b x 9876
g. 9876c = 9876 x c
h. 4567d = d(4567)
i. 999e = 999(e)

24.6 Brackets:

24.6.1 Teachers, now is an opportunity to explain, expansion of brackets.

3(10 + 5 + 2 + 1) = 3 X 10 + 3 X 5 + 3 X 2 + 3 X 1
= 30 + 15 + 6 + 3
= 54
Or 3 X 18 = 54

Similarly a (10 + 5 + 2 + 1) = a X 10 + a X 5 + a X 2 + a X 1
= 10a + 5a + 2a + a
= 18a
LHS = a (18) = 18a

Exercises:

a. Expand: 5 (a + b + c)
b. Expand: a (5 + 3 + 2)
c. Expand: 5 (a + 3)
d. Expand: a (5 + b)
e. Expand: a (a + b + c)
f. Expand: 5 (1 + 2 + 3 + 4+5)
g. Expand: a (1 + 2 + 3 + 4 + 5)
h. Expand: 2 ($x + y + z$)
i. Expand: a ($x + y + z$)

24.6.2 Brackets for division and fractions:

Similarly dividing also

$\frac{10}{3} + \frac{5}{3} + \frac{2}{3} + \frac{1}{3}$ is the same as $\frac{1}{3}$ (10 + 5 + 2 +1)

Or = $\frac{10 + 5 + 2 + 1}{3} = \frac{18}{3} = 6$

Much more clear way will be to write the above as $\frac{(10 + 5 + 2 + 1)}{3}$

i.e. with brackets.
Brackets are used when extra items are there in the numerator.

In algebra $\frac{10}{a} + \frac{5}{a} + \frac{2}{a} + \frac{1}{a}$

$= \frac{1}{a}\ (10 + 5 + 2 + 1) = \frac{18}{a}$

$= \frac{10 + 5 + 2 + 1}{a}$

$= \frac{18}{a}$

Exercises:

a. $\frac{a}{5} + \frac{b}{5} = ?$ For a = 4, b = 1

b. Given that a + b = 10 Find $\frac{a}{5} + \frac{b}{5}$

c. Given that a= 4 find $\frac{5}{a} + \frac{3}{a}$

d. First Simplify $[\frac{4}{a} + \frac{3}{a} + \frac{3}{b} + \frac{5}{b}]$

Then find the value for a =7 and b = 8

24.7 Caution $a \neq b \neq c$
In number system

2 + 3 + 4 = 9 can add up, because they all belong to **ONE FAMILY**.

But 2a + 3b + 4c + a + b + c
Can be written, by grouping together same family members.

= 2a + a + 3b + b + 4c + c
= 3a + 4b + 5c

In the above example, if a, b, c, are substituted by numbers, then they can join.
Eg: Let a = 1, b = 2, c = 3
LHS = 3a + 4b + 5c

= 3 x 1 + 4 x 2 + 5 x 3
= 3 + 8 + 15
= 26

a. After substitution a, b, c etc do not exist any more. They have been converted to numbers.

24.8 Caution $a \neq a^2$
In number system, a number and its square can add up.

E.g.: $3 + 3^2 = 3 + 9$
$= 12$

Or $5 + 2\ (5)^2 = 5 + 2 \times (25)$
$= 5 + 50$
$= 55$

In algebra a, a^2 will remain independent

Thus $a + a^2$ = same only
$x + 2x^2$ = same only

In the above example, if a, x etc are substituted by numbers, then they can be added.
Thus If a = 3, $a + a^2 = 3 + 3^2 = 12$
Thus If $x = 5$, $x + 2x^2 = 5 + 2(5)^2 = 55$

Repeat (a) from sec 24.7

24.8.1 Exercises: Simply

a. a + b + 3a + 4b
b. a – b + 3a – 4b
c. –a + b –3a + 4b
d. a(a+1) + b(b+1) + c(c+1)
e. a(1 + 2 + 3) + b(2 + 3 + 4) + c(3 + 4 + 5)
(f) to (j) If (A) to (E) above, find the values if a = 3, b = 5, c= 7
k. $3^2 + 5^2 + 7^2 = ?$
l. 3 + 5 + 7 =?
m. $(3 + 5 + 7)^2$
n. Which is bigger m or k?

24.9 Caution: No mixing

The idea given in 35.7 is usually explained by resourceful teachers as “apples and oranges” do not mix. i.e. If a is for apple and b is for orange or banana.

“a” are kept in one basket
“b” are kept in another basket

Thus if there are 10 apples in one basket and 6 bananas in another basket.
10a in one and 6b in another.

Now if you empty both of them into a big basket.

(10a + 6b) is there in a big basket.

This is how algebra differs from numbers.

24.10 Students can skip this section (=omit, need not read)

(This Para is for teachers only).

Some times apples and bananas may be grouped into fruits.
In that case a, b both will belong to a big family of fruits.
Then a, b will be called subsets of a set of fruits (call ‘f’ if you like).
This is called **SET THEORY** and practical engineering does not need it.
If discussion comes to a ‘basketful of fruits’, then you can say,

a = f, b (also) = f

$\therefore$ 10a + 6b = 10f + 6f = 16f (This is the basketful of fruits).
In such a case a, b will be called subsets.

24.11 Subtraction or Handling –ve quantities

24.11.1 Subtraction can also be done in the same way as addition.

Remember	5 - 4 = 1	means 1 of (+) remains
	5 – 5 = 0	means nothing remains
	5 – 6 = -1	means 1 of (–) remains

A. This means subtraction is only a method of combining, including the minus sign.
B. In traditional subtraction we handle only 2 items at a time. Now let us see how we can handle many items.

24.11.2 Many –ve numbers

The idea of 24.11.1 helps in handling many numbers at a time.
Thus 100 – 40 – 30 – 10 – 5 – 4 =?
By the traditional concept of subtraction, you have to do one pair at a time.

Thus 100 – 40 = 60
60 – 30 = 30
30 – 10 = 20
20 – 5 = 15
15 – 4 = 11

Forget subtraction; call the problem as a combination of both (+) & (-) numbers.
In that case: (+) is 100

(-) is 40 + 30 + 10 + 5 + 4 = 89
Thus LHS = +100 – 89 = 11
This can also be written as 100 - (40 + 30 + 10 + 5 + 4)
i.e. 100 – 89 = 11

24.12.1 Same as 24.11.1

$5x - 4x = 1x = x$
$5x - 5x = 0x = 0$
$5x - 6x = -1x = -x$

Repeat (A) from 24.11.1
Repeat (B) from 24.11.1

Exercises:
Example: 1234a – 1033a =?
Ans: LHS = a (1234 – 1033)
= a x 201 = 201a

Do:
a. $5243x - 5243x$ =?
b. $243y - 242y$ =?
c. $5243x - 5244x$ =?

24.12.2 This is the same as 24.11.2, but with y added. Now we call it Algebraic quantity.

$100y - 40y - 30y - 10y - 5y - 4y$ =?
i.e. $100y - y(40 + 30 + 10 + 5 + 4)$
i.e. $100y - 89y$
$= 11y$

Exercises:
Example: 1234a – 1000a – 30a – 3a =?
Ans: LHS = 1234a – a (1000 + 30 + 3)
= 1234a – a (1033)
= a (1234 – 1033)
= a (201)
= 201a

This looks long. But doing this way makes a student good at doing algebra and no mistakes. The confidence gained helps. Do the following by steps:

A. $5243x - 5000x - 200x - 40x - 2x - x$
B. $243y - 200y - 39y - y - 2y$
C. 5243a – 5200a – 40a – 4a
D. 8a – 7a + 3a – 2a + 4a – 5a
E. 128a – 127a + 123a – 122a + 124a – 125a

24.13 Activity

24.13.1 Addition with numbers only. Earlier we have prepared bricks (some people call it 'Tiles' – In scrabble game also they call the letters as 'Tiles' – so, we start calling them Tiles). Those tiles were sets of 0 to 9 – i.e., one one-digit number on each tile. Now put –ve numbers also.

Teachers, go back to number bricks you had prepared earlier. Make equal number of negative numbers. Let students play in groups.

a. One from positive set; one from negative set. Put them together; pick the result.
b. 2 from positive set; two from negative set.
c. Some from positive set; some from negative set.

24.13.2 Addition with x, y (Algebra)

Prepare similar sets of x numbers and y numbers.

a. Play the game of (a) (b) (c) above with x set only.
b. Play the game of (a) (b) (c) above with y set only.
c. Play the game of (a) (b) (c) above with x, y, (+), (-) mixed.
d. Now mix number sets also.

For a self – study student, this looks very complicated. Such a student can get the help of teachers / elders / other senior students.

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Chapter - 25 Basic Operations in Algebra - B

25. Basic operations in algebra (Contd.)

We saw addition and subtraction. We also reduced both to joining together of positive and negative numbers or symbols (algebraic quantities). Here we see multiplication & division.

25.1 Observe carefully (better still write down):
10 X 2 = 20
10 X a = 10a
10 X 2a = 20a
10a X 2 = 20a
10 X 2 X a = 20a
20 X a = 20a
20 X 1 = 20 x 1 = 20
20 X 21 = (20 X 21) = 420
20 X 21a = (20 X 21) a = 420a

What is you observe?
These were not given for the sake of finding answers (in students language "doing sums" "Lekkaa Maaduvudhu"). These are given for you to observe the rule.

Rule: Numbers join up. Separately worked out.
Letters (=algebraic quantities) follow.

Convention: 10 x a, 20 x a, 420 x a are written as 10a, 20a, 420a.

Caution: a10, a20, a420 do not exist. They are not written like that. If you want algebraic quantities to be written first, you can write as:
a x10, a x 20, a x 420

25.1.1 Exercises: Say Right / Wrong

a. 5 x b = 5b
b. 5 x b = b5
c. 5 x 20 x b = b100
d. 5 x 20b =5b20

e. 5 x 20b = 100b
f. 25 x 2b = 50b
g. 25 x 2b = 50 x b
h. 25 x 2b = b50
i. 25 x 2b = b x 50

25.1.2 Exercises: Say Right / Wrong. They are the same as (a) to (i) above. Let b = 3, then

a. 5 x b = 5b = 53
b. 5 x b = b5 =35
c. 5 x 20 x b = b100 = 3100
d. 5 x 20 x b = 5b20 = 5320
e. 5 x 20 x b = 100b = 300
f. 25 x 2b = 50b = 150
g. 25 x 2b = 50 x b = 150
h. 25 x 2b = b50 = 350
i. 25 x 2b = b x 50 = 150

25.1.3 Activity

Let each student pick bricks from the number sets, add X and play this game (At least one problem per student). (Brick = Tile in number set).

25.2 Division

Just like you can divide a number by a number, in the case of algebraic quantities, you can (say x):

a. Divide x by any number
b. Divide any number by x
c. Divide x by x itself
d. Divide x by any other y

Let us do step by step (i), (ii) first.

25.2.1 $\frac{10}{2} = 5$ (Same as 10 ÷ 2 = 5)

$$\frac{10\ a}{2} = 5\ a$$

25.2.2 Activity

A. Play a game of division using only integers (go back to the chapter on fractions and learn how to do).

Make 'Tiles':	10	11	……	20	2 digit numbers.
Make 'Tiles':	1	2		9	1 digit numbers.

One set of students will pick only 2 digit numbers. They are numerators. (= Top numbers).
Second set of students will pick only 1 digit number. They are denominators. (=Bottom numbers).

Answers will be written down and checked by the teacher (or other students).

B. Now, play the same game with x. To play the game let one group have 2 digit numbers – the other group 1 to 9. Add x to the 2 digit numbers and let it be the numerator. Divide by the one digit number. Display the results. Let there be as many results as there are students. Teachers may go and check the results, one by one. (You may need x bricks, as many as the number of students.

C. Now let everyone be the numerator. Denominator will be a sweeping (mobile). 10 will come and stand as denominator. While leaving, it will leave a print (=point) on the numerator. Thus producing decimal numbers.

Later add X to the numerator and play the same game. Generate decimals.

How to play:
Starting: 5, 9, 14 standing
Decimal denominator comes around.
During game $\frac{5}{10}$, $\frac{9}{10}$, $\frac{14}{10}$
10 leaves.
End of game .5, .9, .14
When X is added to the numerator.
End of game .5X, .9X, .14 X

D. Vary (c) above to include 100, 1000 as denominators. First play with numbers only. Later add X to the numerator.
You will get .05X, .005X etc

25.3 Multiplication: Many Numbers

a. Drill multiplication of 2 numbers
Eg: 2 x 3 = 6 2a x 3 = 6a
2 x 3a = 6a
2 x 3 x a = 6a

b. Go to 3 numbers
2 x 3 x 4 = 12
2a x 3 x 4 = 12a
2 x 3a x 4 =12a
2 x 3 x 4a = 12a
2 x 3 x 4 x a = 12a

c. Exercises:
1. X x 15 x 2 =? 2. 15 x X x 2 =? 3. 2 x 15 x X =?
4. 1 X 15 x 2X =? 5. 2 x 15X x 3 =? 6. 4X x 15 =?
7. 15X x 4 =? 8. 15 (X +4) =? 9. X + (15 x 4) =?

25.4 Division: Many Numbers

25.4.1 a. Drill division of 2 numbers

$$\frac{6}{2} = 3 \qquad \frac{6a}{2} = 3a$$

b. Make 2 numbers in numerator & one number in denominator.

$$\frac{6 \text{ x } 5}{2} = 3 \text{ x } 5 = 15$$

$$\frac{6 \text{ x } 5 \text{ x } a}{2} = 15a$$

$$\frac{6a \text{ x } 5}{2} = 15a$$

Exercises:

1. $\frac{x \text{ x } 15 \text{ x } 2}{15} = ?$ 2. $\frac{2 \text{ x } 15 \text{ } x}{15} = ?$ 3. $\frac{2 \text{ x } 15 \text{ } x}{2} = ?$

4. $\frac{15 (x + 4)}{15} = ?$ 5. $\frac{15 (x + 4)}{5} = ?$ 6. $\frac{x (15 + 4)}{19} = ?$

7. $\frac{x + (15 + 4)}{x + 19} = ?$

25.4.2 Many items in both numerator and denominator.

a. $\frac{1 \text{ x } 2 \text{ x } 3 \text{ x } 4 \text{ x } 5}{2 \text{ x } 3}$ = 1 x 4 x 5 = 20

b. $\frac{11 \text{ x } 13 \text{ x } 15 \text{ x } 17}{11 \text{ x } 17 \text{ x } 5} = \frac{13 \text{ x } 15}{5} = \frac{13 \text{ x } 3 \text{ x } 5}{5} = 13 \text{ x } 3 = 39$

c. $\frac{22 \text{ x } 26 \text{ x } 30 \text{ x } 37}{11 \text{ x } 13 \text{ x } 15} = 2 \text{ x } 2 \text{ x } 2 \text{ x } 37 = 8 \text{ x } 37 = 296$

d. $\frac{22 \text{ x } 26 \text{ x } 30 \text{ x } 37}{44 \text{ x } 13 \text{ x } 60 \text{ x } 74} = \frac{1}{2} \text{ x } 2 \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{2 \text{ x } 2} = \frac{1}{4}$

e. Now add x (or any algebraic quantity anywhere) (only one at a time)

1. $\frac{1 \text{ x } 2 \text{ x } 3 \text{ x } 4x}{2 \text{x } 3} = 4x$ 2. $\frac{11 \text{ x } 13 \text{ x } 15 \text{ x } 17}{13 \text{ x } 11 \text{ x } 17 \text{ x } x} = \frac{15}{x}$

3. In (2) above add y to numerator. Ans: Will be 296y.

If (c) above, add y to denominator. Ans: will be $\frac{296}{y}$

Exercises:

1. Students, you can make your own problems. [Help: use numbers only in the numerator. Use numbers and letters in the denominator. Do vice versa (=ulta)]

2. Simplify:

a. $\frac{9+8+7+6}{15x}$ b. $\frac{(9+8+7+6)x}{15}$ c. $\frac{(9+8+7+6)x}{15x}$

d. $\frac{9x+8+7+6x}{15x+15}$ e. $\frac{9+8+7+6}{15x}$ f. $\frac{(5+8)+10}{100x}$

g. $\frac{(5+8)x+10x}{50x}$

25.5 Division as fractions.
We have seen addition of fractions. Let us revise by doing some exercises.

25.5.1 Exercises: Say True / False

a. $\frac{1}{4}+\frac{2}{4}=\frac{3}{8}$ b. $\frac{1}{4}+\frac{2}{4}=\frac{2}{4}=\frac{1}{2}$ c. $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$

d. $\frac{2}{11}+\frac{3}{11}+\frac{5}{11}+\frac{1}{11}=\frac{11}{44}=\frac{1}{4}$ e. $\frac{2}{11}+\frac{3}{11}+\frac{5}{11}+\frac{1}{11}=\frac{11}{11}=1$

25.5.2 Recall (= remember, think about it again) the rule. "When the denominator is the same, numerators can be added".
Now apply this to the problems above and see how you fared (= did well or not).

25.5.3 Apply the above rule when the fraction contains letters instead of numbers.

a. $\frac{1}{n}+\frac{2}{n}=?$ Ans: LHS = $\frac{1+2}{n}=\frac{3}{n}$

b. $\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\frac{5}{n}=?$ Ans: LHS = $\frac{1+2+3+5}{n}=\frac{11}{n}$

Exercises:

a. $\frac{2}{x}+\frac{3}{x}=?$ b. If x =5, same =?

c. $\frac{1}{n}+\frac{2}{n}+\frac{3}{n}=?$ d. If n = 6, same =?

e. $\frac{1}{d}+\frac{2}{d}+\frac{3}{d}+\frac{4}{d}+\frac{5}{d}+\frac{6}{d}+\frac{7}{d}+\frac{8}{d}+\frac{9}{d}=?$ f. If d = 15, same =?

g. If d = 5, same =? h. If d = 3, same =? i. If d = 45, same =?

j. $\frac{a}{5}+\frac{b}{5}+\frac{c}{5}+\frac{d}{5}+\frac{e}{5}=?$ k. If a = b = c= d = e = 2, same =?

l. Instead of 5, substitute 'n', same=?

25.6 Teachers! Tell the students that mixed denominator is not allowed for simplifying.

E.g.: a. $\frac{10}{5+a}$ remains as such.

b. $\frac{10}{1+4}$ Becomes $\frac{10}{5}=2$ OK

c. $\frac{10\,a}{1+4}$ = $\frac{10a}{5}$ = 2a OK

d. $\frac{10+5}{a}$ Becomes $\frac{15}{a}$ OK

e. $\frac{10+5a}{5+a}$ ≠ $\frac{10}{5}+\frac{10}{a}$ This is wrong.

f. $\frac{10}{5+a}$ ≠ $\frac{10}{5}+\frac{5a}{a}$ Wrong.

(5), (6) above cannot be simplified. If we know a = same number, then all of it will join to become numbers.

25.7 Note to teachers: in the following examples, we assume some knowledge, i.e., squares, cubes etc., i.e., Simple Indices. Later, there will be a chapter on square and square roots. A few points here is worth learning before we handle mixed quantities (i.e., mixture of numbers and algebraic quantities).

25.7.1 Please note: Multiplication tables can be generated by addition.
i.e., 4 x 5 = 20 means
4 + 4 + 4 + 4+ 4 …… (5 times) = 20
or 5 + 5 + 5 + 5 + 5 (4 times) = 20
This is the reason we stated earlier, repeated addition is multiplication.

25.7.2 Now consider (a) 3 + 3+ 3 and (b) 3 x 3 x 3
(a) = 9 = 3 times 3 = 9
(b) = (3 x 3) x 3 = 9 x 3 = 27
(b) is different from (a)
(b) is repeated multiplication.

4 + 4 + 4 ≠ 4 x 4 x 4

LHS = 12 = 3 x 4
RHS = 64

Symbol of repeated multiplication is called Index or Power*. It is written as a small size number on the right hand side top of the number.
[* strict maths calls this "Exponent"]

Thus, $2 \times 2 \times 2 = (2)^3$ or 2^3
$3 \times 3 \times 3 \times 3 = (3)^4$ or 3^4
$4 \times 4 = (4)^2$ or 4^2

25.7.3 How to read (Indices)
When x, y, a, b, alone occurs many times then use "Power".
$y \times y = y^2$ (read as 'y square')
$y \times y \times y = y^3$ (read as 'y cube')
$y \times y \times y \times y = y^4$ (read as 'y to the power of 4')
[4 onwards 'to the power of' is used while reading out]

25.7.4 How to write (Indices)
a. b x b x b …..8 times is written as $(b)^8$ or b^8 (8 is small).
b x b x b …..n times in $(b)^n$ or b^n.

b. If (x+1) is multiplied by itself, (x + 1) x (x + 1). This can also be written as (x + 1).(x + 1) [x or .].
Both mostly written as (x + 1) (x +1).
Since the same thing occurs 2 times (x + 1) (x + 1) is the same as (x + 1).
Similarly $(a + b + c)^4$ or $(a + 5)^n$ etc.,

25.8 Multiplication of mixed items.
A. Worked examples:
a. 2 x 3 =6
b. 2a x 3 = 6a
c. a x a = a2
d. 2a x 3a = 6a2
f. 2a x 3a x 4a x 5a = (2 x 3 x 4 x 5) (a x a x a x a) = $120.a^4$

B. Exercises: x y

1. 2x x 3 =? 2. 2x x 9x =? 3. 1.5x x 4 =?
4. .9x x 10 =? 5. 2x x 2x x 2x =? 6. y x y x y …(10 times) =?
7. d + d+ d + d + …. (10 times) =? 8. b x b x b …. (n times) =?
9. x x x x (m times) =?

25.9 Division: Mixed quantities.

A. Worked Examples:

a. $\frac{2 \text{ x } 3}{3} = 2$ b. $\frac{2 \text{ x } 3 \text{ x } 4 \text{ x } 5}{2 \text{ x } 3} = 4 \text{ x } 5 = 20$

c. $\frac{2 \text{ x } 3 \text{ x } 4 \text{ x } 5 \text{ x } a}{2 \text{ x } 3} = 4 \text{ x } 5 \text{ x } a = 20a$ d. $\frac{2 \text{ x } 3 \text{ x } 4 \text{ x } 5 \text{ x } a}{2 \text{ x } 3 \text{ x } a} = 20$

e. $\frac{2 \text{ x } 3 \text{ x } a3}{3a} = 2 \text{ x } 3 \text{ x } a \text{ x } a \text{ x } a = 2 \text{ x } a \text{ x } a = 2a^2$

B. Exercises:

a. $\frac{a \text{ x } 2 \text{ x } 3a \text{ x } 4 \text{ x } 5a}{2 \text{ x } 3} = ?$ b. $\frac{11 \text{ x } 13 \text{ x } 15a \text{ x } 17a}{11 \text{ x } 17 \text{ x } 5}$ c. $\frac{22a \text{ x } 26a \text{ x } 30a \text{ x } 37a}{11a \text{ x } 13a \text{ x } 15a} = ?$

25.10 Concept of reciprocal.

Definition: Reciprocal of a number = $\frac{1}{\text{the number}}$

This is simply to be accepted as a word and its meaning.

25.10.1 This, reciprocal of 10 = $\frac{1}{10} = .1$

Reciprocal of 100 = $\frac{1}{100} = .01$

Reciprocal of 3 = $\frac{1}{3} = 0.33$

Reciprocal of $\frac{1}{3}$ = $\frac{1}{1/3} = 3$

Reciprocal of .2 = $\frac{1}{.2} = 5$

25.10.1 Exercises: Write down the reciprocal of

a. 5 b. 50 c. 500 d. 0.1 e. 0.01 f. 0.001

g. 8 h. $\frac{1}{8}$ i. $\frac{8}{3}$ j. $\frac{3}{8}$ k. $\frac{8}{100}$ l. $\frac{100}{125}$

Why the idea of reciprocal?

It helps to remove the fear of 'DIVISION' more importantly, if you know there is a reciprocal, you need not have any division any more. Only multiplication will do. (Will do = is enough, is sufficient).

Example:

a) $\frac{8}{3}$ = 8 x $\frac{1}{3}$ = 8 x ($\frac{1}{3}$)

b) $\frac{3}{4x7}$ = 3 x ($\frac{1}{4}$) x ($\frac{1}{7}$) i.e., 3 multiplications

Exercises:

a. Given $\frac{1}{2}$ = .5, find $\frac{x \text{ X } y}{2}$ where $x = 10$ $y = 6$

b. Find $\frac{axb}{125}$ where a = 12; b = 5

Given: $\frac{1}{125} = .008$

c. Given $\frac{1}{333} = .003$, find $\frac{66 \text{ x } 99}{333}$

(Clue: Do approximately; do not use calculator)

We know a Addition & Subtraction are Not two different actions. It is only one action i.e., subtraction (with a sign -) is a special case of addition of negative numbers. This we explained with a ball & hole concept.
Now we know that Multiplication and Division are not two different actions. It is only one action i.e., of multiplying with a reciprocal is a special case, also called division. In this system where do +ve & -ve numbers come in?

25.12 Let us clarify to the students one of the problems in algebra i.e., + & -

<u>Addition Rules:</u>

Rule 1. + (a) + (b) Ans: + ve (add both)

Rule 2. + (a) – (b) Ans: + ve if a> b
Do (a - b)
It is – ve if b > a
Do (b – a)

(Rule 2 is often stated as: subtract smaller from bigger and the final sign will be that of the bigger)

Rule 3. – (a) – (b) Ans: – (a + b)
- ve (add both)

[Rule 3 often stated as: Add the numbers & final sign is – ve]

Multiplication Rules:

Rule 1. (+a) x (+b) = +ab. +ve

Rule 2. (+a) x (-b) = -ve (a x b)
(-a) x (+b) = -ve (a x b)

Rule 3. (-a) x (-b) = +ve (a x b)

In students language

(+) x (+) = +	(-) x (-) = +
(+) x (-) = -	

This rule works. Follow it.

[For advanced – level teachers, there is an appendix on it]

Chapter - 26 Equations

26. Equations:

Equations are very useful in algebra. They are useful in all branches of science. Engineering is full of equations. Economics (selling, buying, estimating) transactions use equations. So equations are everywhere.

26.1 How to convert simple statements onto equations?

"I bought a bicycle" for 2000 rupees.

i.e. my bicycle cost me Rs. 2000
i.e. cost of my bicycle is Rs. 2000
i.e. cost of my bicycle = Rs. 2000

Let c be the cost of my bicycle c = Rs. 2000

Suppose I bought a new Atlas cycle. Then I can even say c = Rs. 2000 where c = cost of an Atlas cycle.

26.2 Simple statements:

Grandfather "Bachcha, I am three times older than you".

i.e. Grandfather's age is 3 times that of Bachcha's age.
i.e. Grandfather's age is 3 times that of Bachcha's age.
i.e. Grandfather's age = 3 x Bachcha's age
i.e. $y = 3x$

Where

$y = ...$
(Let students fill this up)
$x = ...$

Teachers should draw the attention of the students. The brevity and clarity of the equation given in 26.1 & 26.2. Try some of the statements.

a. My house is only 10 minutes walk from your house.
b. New road can take from Mysore to Bangalore in 2 hours.
c. I am taller than you by 6 inches.

[Note for teachers: very few maths books include problems of this type. CBSE books are helpful. Formulas found in economics, commerce, business organization high level books (eg: B.Com, BBM, MBA etc) are similar to what is written here). But we do not need all those for teaching this chapter. More knowledge helps]

26.3 Try some puzzles

a. Father is twice as old as the son. If son's age is 25 what is the father's?
b. In (a) above; the daughter is 3 years younger than the son. The daughter is 3 years older than me. My age is 15.What is the father's age
c. Father is four times as old as his son. In twenty years, son will be half his father's age. What are their ages?
d. Pens and notebooks were bought. 10 pens and 2 notebooks cost Rs. 100. What are their prices?
e. In (d) above for the same money I could have bought 5 pens and 6 notebooks. What are their prices?

[Help: Earlier sections had only equations No maths. The above can be converted into equation, and solved also.
In (d) above many answers are possible.

In (e) the same is restricted to one value.

26.4 Activity
Every shopkeeper, vegetable vendor, storekeeper, bus conductor, bank manager uses mental algebra and equations. Let the students try some examples.
Students can play games using this idea. One set can use actual money, another only equations**.**

26.5 Exercise:
If you are drawing Rs.1000 from a bank, how many different ways could the cashier pay you?

Assume
a = One thousand rupee note
b = Five hundred rupee note
c = One hundred rupee note
d = Fifty rupee note
e = Ten rupee note
f = Five rupee note
g = One rupee note / coin

Many answers could be generated. Let the whole class try it come out on equation form.

Eg:
1000 = 1000g
1000 = a
1000 = b + 5c

26.6. Students, recall (= remember, think again) that we started with the process of substitution. In a blank space (as in an application form) you write your name, others write other names. Here you can write x. (or any letter).Then it looks like Maths. It is called algebra. It is simple. When something is not known, you call it x. This is how it started. Then the habit spread. Now any letter can be used to represent ‘ the unknown ’.

26.6.1 Equation form of normal statements.

a. There are some fruits in the basket.
You can take half of them.

Let fruits in the baskets = x

Your share = $\frac{x}{2}$

b. Question: If there were 8 fruits, how many will you take?

Ans: x = 8, your share = $\frac{x}{2}$

$= \frac{8}{2}$

= 4

c. Question: In (a) above, your share was 6 fruits. How many were in the basket?

Ans: Your share $= \frac{x}{2}$

$\frac{x}{2} = 6$

$x = 12$

Exercises A: Do As Per (a) above.

1. I have some money in my account.
2. I owe you some money

3. He commutes (= travels for work) up and down & covers some Km per day.
4. Do you eat only this many idlies? I eat 3 times that many.
5. You take this much time to do this work. Nirmala can finish in half the time.
6. With the money you pay for one person in Hotel Paradise, you can give lunch to 4 persons in Darshini.
7. In “SALE” time, we give 10% discount on all items of purchase.

Exercises B: Do as per 26.6.1 b and 26.6.1 c (See exercises A (1) to (7) above)

1. Minimum balance required is Rs. 5000. I have Rs. 500 in the bank. How much money (maximum) I could draw (= take out)?
2. I owe you Rs. 100.I’ll return it after 1 year with 10 % interest. How much will I pay you next year?
3. If he travels 200 km/day. What is the distance between his houses office?
4. I think 8 idlies will be needed for both of us. How many idlies each one eats?
5. Nirmala was given 50%(= half) the work. She finished her work in 1 day. How long will you take for your work? What was the total time taken for the work?
6. Darshini gives lunch for Rs.15 What is the bill for one person in Hotel Paradise?
7. MRP written on some items are as given below.

   Salt 1kg packet Rs. 6
   Dal 1kg packet Rs. 60
   Sugar 5kg packet Rs. 120
   Electronics items Rs5000.

   You bought 1 of all these items. How much (total) money did you save?

Exercises C: Combination of exercises A (5) & B (5)

1. K is a slow worker. N can do the work in half the time (i.e., compared to K). 300 pages of some work is to be done. If the work is shared between K & N and if you would like the work to be finished simultaneously (= at the same time), how many pages will you give to N, and how many to K?
2. If the work can be finished in 10 days, how will the work be shared on daily basis?

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Chapter - 27 **Working with Equations - A**

27. Working with Equations:
Simple symbols (algebra) and equations go together. They make many activities simpler, easier and more accurate. Many guessing games could be avoided by using proper equations.

27.1 Some Symbols How to write:

$x = y$	5 = 5	10 = 10	12345 = 12345	$x > y$	5 > 4
10 > 9	12345 > 12344	$x < y$	5 < 6	10 < 11	12345 < 12350
$x \neq y$	$5 \neq 6$	$5 \neq 4$	$5 \neq 22000$		

These are very basic symbols. The most important is the first one (=).
There are some others also $\geq$, $\leq$, $\not>$, $\not<$ etc.

Some others are A > x < B
They are not very important.

27.1.1 How to read:

= ‘is equal to’ OR
Equal to OR
Equal to OR
Equals.

$x > y$	x	(is) greater than y (is) optional.
$x < y$	x	less than y
$x \neq y$	x	not equal to y
$x \geq y$	x	greater than or equal to y
$x \leq y$	x	less than or equal to y
$x \ngtr y$	x	not greater than y
$x \nless y$	x	not less than y
$x \approx y$	x	approximately equal to y

27.2 Simplification: How to write:
Writing (=) sign.

2(3 + 2) + 5 (4 - 3) + 25 =?
2 x (5) + 5(4 – 3) + 25 =?
0 + 5 (1) + 25 =?
10 + 5 + 25 =?
15 + 25 =?
$\therefore$? = 40

This is a poor way of writing; even with so many steps. Instead write:

2(3 + 2) + 5 (4 - 3) + 25
= 2 x (5) + 5(4 – 3) + 25
= 10 + 5 (1) + 25
= 10 + 5 + 25
= 15 + 25
= 40

Therefore put (=) sign one below another and each step follows the previous step.

27.2.1 EXERCISE

Follow the step-by-step principle and the Practice of writing = sign one below the earlier. Simplify

a. Product = 1 x 2 x 3 x 4 x 5 x 6
[Clue: 5 steps are needed]
b. Product = 2 x 5 x x x 3 x y x 4
c. Result = 2 (5+ x) + 3(y +4)
d. Result = 2 (5+ x) + 3(x +4)
e. Result = 3 (x +4) - 2(5+ x)
f. Result = 2 (5+ x) - 3(x +4)

27.3 Cost of one dozen bananas = Rs. 24

Cost of one banana = $\frac{24}{12}$

Cost of one banana = $\frac{2X\,\cancel{12}}{2\,X\,\cancel{6}}$

Cost of one banana = 2

What is wrong here? Waste of energy etc.

Therefore write cost of one banana = $\frac{24}{12}$

$$= \frac{2 \text{ X } \not{12}}{2 \text{ X } \not{6}}$$

= 2 (= sign one below the previous)

27.3.1 Students! From the example given above what did you learn?
Ans: you learnt how to do the problem (lekkaa maaduvudhu) and also how to write it. Until you learn how to neatly write & logically arrange your thoughts, do not go for shortcuts. [Ask for forgiveness from Parisara Devathe for wasting Paper-Use slates if take long time to learn]

27.3.2 Exercises:

Do step by systematically:

a. Cost of 1 kg rice is Rs.38 (as on May, 2009 Mysore)

i) What is the cost of 5 kg?

ii) What is the cost of 300 grams?

b. 5 liters of branded oil costs Rs 420, Loose oil ½ liter bought cost was Rs. 40. Which is cheaper?

27.4. LHS = RHS is the Rule.

An equation has LHS & RHS.
LHS must always be equal to RHS.
We know that **10 + 5 = 15.**
The equation is OK if we ADD the SAME thing to both LHS & RHS of the equation.

Thus **10 + 5** + 20 **= 15** + 20
Or 20 + **10 + 5 =** 20 + **15**

Similarly **10 + 5** + 123456 **= 15** + 123456
123456 + **10 + 5 =** 123456 + **15**

Similarly **10 + 5** + 1 + 2 + 3 + 4 + 5 **= 15** + 1 + 2 + 3 + 4 + 5
(i.e. add any number of items).

Similarly $\mathbf{10 + 5} + x \mathbf{= 15} + x$
$\mathbf{10 + 5} + (x^2 + x + y^2 + y + 25) \mathbf{= 15} + (x^2 + x + y^2 + y + 25)$

27.4.1 Exercises:

You are given $\boxed{x = y}$
State which of the following is True.

a. $x + 2 = y + 2$
b. $x + 2 = y + 20$
c. $x + 2 = 2y + 1$
d. $x + 2 = y + 5 - 3$
e. $2x + 4 = 2y + 4$
f. $2x + 4 = y + 8$
g. $x \times x = y \times y$
h. $x + a = y + a$
j. $x + a + b = y + a + b$.

27.4.2 Subtraction is the same as adding a negative item. So the above rule applies to (-) minus also.

$\therefore$ Given that A = B

A+ (anything) = B + (anything)

A – (something) = B – (something)

Exercises: You are given $x = y$ State which of the following is True.

a.	$x - 2 = y - 2$	b.	$x - 2 = y - 20$
c.	$x - 2 = 2y - 1$	d.	$x - 2 = y - 5 – 3$
e.	$2x - 4 = 2y - 4$	f.	$2x - 4 = y - 8$
g.	x x x = y x y	h.	$x - a = y - a$
j.	$x - a - b = y - a - b$.		

27.5 **FOR TEACHERS**.
This section is for teachers. Students may try to read. If it is difficult, skip and go to 27.6.

27.5.1 Given that A = B
We know A + 10 = B + 10
Is it correct A +10 = B + 8 + 2 It is OK because 10 = 2 + 8

Rules1: Adding the same to LHS & RHS is ok

Rules2: Adding equal items to LHS & RHS is ok

27.5.2 Same as above with (- ve).
Given that A = B

A – (an item) = B – (an item)

Also A – (item) = B – ($\frac{1}{2}$ item + $\frac{1}{2}$ item)

√ A – (10) = B – (10)
√ A – 10 = B – (5 + 5)
√ A – 10 = B – (2 + 8)
√ A – 10 = B – (5 + 2 + 3)
X A – 10 = B – 5 + 2 + 3 Wrong

27.5.3 General rule:

A + x = B + y
A - x = B - y

A = B and $x = y$

(Teachers, give large number of examples for this and See if the written matter is OK).

27.5.4 Some more examples (for 27.8).

a. 100 = 50 x 2

8 = 4 x 2

100 + 8 = 108 = (50 x 2) + (4 x 2) √ OK
100 – 8 = 92 = (50 x 2) – (4 x 2) √ OK
Now try 100 + (50 x 2) = 8 + (4 x 2) NOT OK

b. Why? If A = B and $x = y$

A + 1234= B + [?] only 1234 or its equals.

Similarly A + x = B + [?] x itself

= B + x or items which are equal to x

= B + y because $y = x$

In the above example, even it $x = y$

If you do A + B = [1] [2]

[1] Can only be something equal to A

[2] Can only be something equal to B

A + B = $x + y$ is wrong NOT OK
Because A $\neq x$; B $\neq y$

27.6 For STUDENTS & TEACHERS
This rule is very simple. It says, you can add anything to LHS of an equation, equality will be true if you add the same thing to RHS also.
Same here applies to equals also.

Thus if

A = B
A+ x = B + x
A+ y = B + y
A - x = B - x
A - y = B - y

If $x = y$, then A+ x = B + y .
A - x = B - y .

If $x \neq y$ then this rule does not apply.

27.6.1 Exercises: State true/ false showing steps to prove your answer.

i. x + a – b = x – (b – a) ii. x + 100 = x + (50 x 2)

iii. $x + y$ +2z = x +2(y +z) iv. x + c = $x + \frac{c}{2} + \frac{c}{2}$

vi. x -8 – 8 b= x - 8(b+1) v. 4(x + a) = 4 x +4 a

27.7 For teachers only:
Self – learning students can skip this and go to 27.8.

27.7.1. Now try 100 = 50 x 2; 10 = 5 x 2

$$\frac{100}{10} = \frac{50 \times 2}{5 \times 2} = 10$$ OK

Also $$\frac{100}{50 \times 2} = \frac{10}{5 \times 2} = 1$$ √ OK

How? It is because all are = 1 **Special Case**

Some more examples (27.9 c)

a. Let 100 = 50 x 2; 8 = 4 x 2

Now try 100 x 8 = (50 x 2) x (4 x 2)

800 = 800 √ OK

b. Again try (100) x (50 x 2) = 8 x (4 x 2) NOT OK

c. ∴ If A = B; $x = y$
A x x = B x y OK

$A \times B \neq x \times y$

d. In (c) above

If A = B, $A \times B = (A)^2 = (B)^2$ OK √

If $x = y$, $x \times y = (x^2) = (y^2)$ OK √

And $Ax = By$ OK √

Also $A \times y = B \times x$ OK √

Also $\frac{A}{x} = \frac{B}{y}$ OK √

Also $\frac{A}{y} = \frac{B}{x}$ OK √

e. Thus $\frac{A}{x} = \frac{B}{y} = \frac{A}{y} = \frac{B}{y}$ all OK

$A \times x = B \times y = A \times y = B \times y$ all are OK

Not $A \times B = x \times y$ Not OK

27.8 For both Students & Teachers.

Rule:

If A = B

$A \times y = B \times y$

$\frac{x}{A} = \frac{x}{B}$, $\frac{y}{A} = \frac{y}{B}$

$A \times x = B x$

$\frac{A}{x} = \frac{B}{x}$, $\frac{A}{y} = \frac{B}{y}$

Condition = $x \neq 0$, $y \neq 0$

Sub rule (Corollary)

If A = B and $x = y$

$A \times x = B \times x = A \times y = B \times y$

$$\frac{A}{x} = \frac{B}{x} = \frac{A}{y} = \frac{B}{y}$$

Caution: Even if $x = y$ & A = B

$A \times B \neq x \times y$

Funny: But $\frac{A}{B} = \frac{x}{y}$, why?

(If A = B, $\frac{A}{B} = 1$ Similarly $\frac{x}{y} = 1$

These are called trivial result)

27.8.1 Activity.

a. Use addition rule to find x in the equations

1) $x - 5 = 10$ x =? 2) $5 - x = 0$

b. Use subtraction rule.

1) x + 5 = 10 x =? 2) 10 - 2x = 0

27.9. Working with equations:
All the four basic arithmetical operations (i.e., -, +, x, ÷) could be done with equations.

27.9.1 Note that 4 = (3 + 1)
Now 40 = 4 x 10 i.e. (3 + 1) X 10 = 30 +10 = 40

∴ If A = B

A x 10 = B x 10 and A X (….) = B X (….)

What is not allowed: (….) cannot be zero.

27.9.2 Similarly division

40 = (3 + 1) X 10

$$\frac{40}{10} = 4 \quad \frac{(3+1)}{10} = 4$$

∴ If A = B

$$\frac{A}{x} = \frac{B}{x} \qquad \frac{1}{x} = (A) = \qquad \frac{1}{x} = (B)$$

27.10 Exercises :

Worked example: Solve x - 20 = 10
[Solve means, find the value of x]

x - 20 = 10 Add 20 to each side
i.e. x - 20 + 20 =10 +20
i.e. x + 0 = 30
i.e. x = 30. Ans

SHORTCUT Method
(a -ve quantity can go to other side but becomes +ve. ie -LHS $\rightarrow$ + RHS.
ie.+ RHS $\rightarrow$ - LHS.)

This way x - 20 = 10
i.e. x = 10 +20
= 30 Ans

Do (i.e. Solve the equations :)

i. $x + 4\frac{1}{2} = 10$

ii. I bought something and gave Rs.10, I got back Rs.4.50 what was the cost of the item bought?

iii. I bought $\frac{1}{2}$ kg of onion, and gave Rs. 10, I got change of Rs.2.50. what was the cost of onion per kg? (Mysore, May09)

iv. A party went to GRS (Fun Park). 5 adults and 10 children were there in the group. Children allowed for $\frac{1}{2}$ the money. A total of Rs.2000 was paid for the entry tickets. What was the cost of adult ticket? Child's ticket?

v. $30x + 50\left(\frac{x}{2}\right) = 110 \quad x = ?$

[Clue: (v) helps to do (iv)
(i) helps to do (ii)

vi. A bus was arranged for a marriage party. It was only for 55 persons. 70 persons came in. Nobody wants to be left. How many autos will you arrange? [clue for foreigners: auto will take only 3 persons/less]

vii 1 bus and 5 autos arrived. A total of 50 persons came in. What was the capacity of the bus?
[Clue: (vi) & (vii) are similar. For some student, one may be easier than the other, do that first]

27.11 Shortcuts

27.11.1 Worked examples:

1. Given that $2x = 220 \quad x = ?$

Divide $\frac{2x}{2} = \frac{220}{2}$ i.e. $x = 110$

Here we divided both LHS & RHS by 2.

Shortcut: if $ax = b$ (a of LHS becomes $\frac{1}{a}$ on RHS)

$$x = \frac{b}{a}$$

$2x = 220$

$$x = \frac{220}{2}$$

$= 110$ Ans

2. Suppose the question was $25x = 225, \quad x = ?$
Let us do as in the example above

Method 1: $25x = 225$

$$\frac{25x}{25} = \frac{225}{25}$$

$$x = \frac{225}{25} = 9$$

Looking at the number 25 on LHS we can do by another method also, i.e. multiplying.

Method 2: $25x = 225$

$4 \times 25x = 4 \times 225$

$100x = 4 \times 225$

$= 900$

$$\frac{100x}{100} = \frac{900}{100}$$

$$x = 9$$

Method 2 appears to be longer, but if you really do the divisions, second method is easier. These methods help us when we have large number or fractions.

27.11.1 Summary:

In method 1, dividing LHS & RHS by the same number is OK.
In method 2, multiplying LHS & RHS by the same number is OK.

27.11.2 Rule:

The methods given above are sometimes shown as transferring to the other side. The shortcuts are multiplying factor on LHS goes as dividing factor on the RHS & vice versa.

$\therefore$ If A X x = B $\quad x = \frac{B}{A}$

If $\frac{x}{A}$ = B, $\quad x$ = B X A

27.11.3 Exercises: Using shortcuts given above, solve:

a. $10x = 100$ x =? b. $\frac{x}{10} = 100$.

c. $125x = 900 + 100$ d. $125x = 250$.

e. $100x = 250 - 25x$ f. $200x = 1000 + 75x$.

g. $\frac{1}{3}x + \frac{1}{2}x = 25$ h. $\frac{1}{3}x - \frac{1}{2}x = 25$

i. a= 2b, b = 2c ; c = 5 a = ?

j. Taruna is twice as old as Beti. Beti is twice as old as Pappu. If Pappu's age is 5 years, how old is Tarun? [Clue: If you can solve (i), you can do (j)

k. If (j) above, Naani is 3 times as old as Taruna, What is the age difference between Naani and Pappu?

27.12 Rules for solving equations:

All the rules put together

If $x + A = B$, $\quad x = B - A$
If $x - A = B$, $\quad x = B + A$
If x X A = B, $\quad x = \frac{B}{A}$
If x X $\frac{1}{A}$ = B, $\quad x$ = B X A

27.13 For Teachers:

Teachers! Simple questions using the above rules could be generated by the students themselves & given to other students. Let them work in groups.

Many maths, puzzles, funs, Sunday magazine sections use these techniques. It will be very easy to find problems by yourselves; or if you just look around.

27.14 Exercises:

a. Ramu is 5 years elder to Radha. If Ramu is your age, What is Radha's age?

b. I gave away 5 rupees each to all the students in the class. The class had 40 students. I was left with 50 rupees. How much money did I have?

c. A player in the casino put some money in the first game. He won & got double the amount. He played like this 4 times. Each time he got double the amount. He added 40 rupees and paid a bill of Rs.200. How much money did he start with?

d. A businessman got a quintal of rice for 1500 rupees and sold in retail at the rate of 21 rupees per kilo. What was his profit? (=Total money earned)? What is his profit per kg? What is his percent profit?
[Help: 1 quintal = 100 kg]

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Chapter - 28 **Squares & Square roots**

28. Squares & Square roots:
A number multiplied by itself is its square. The reverse of it is the square root.

28.1 How to write:

$a \times a = a^2$ (small 2 on the right top)
$5 \times 5 = 5^2$ (small 2 on the right top)
a^2 and $(a)^2$ are the same
6^2 and $(6)^2$ are the same

Brackets must be used when more than one item is squared (= multiplied by itself)
Thus $(5a) \times (5a) = (5a)^2$
Writing this as $5a^2$ is wrong.
Similarly $(ab) \times (ab) = (ab)^2$ writing this as a^2b, ab^2 are wrong.
Similarly $(x + 5)^2$,$(x + y + 2)^2$ ok.
Similarly $(a + b)^2$,$(a – b + 5)^2$ ok.

28.2 How to read:

5^2 is read as " FIVE SQUARE "
[Note: some read as ' Squared ' this assumes a passive voice idea of English grammar, meaning somebody came & squared it. This is Not Important
You can read either way.]
When brackets are there read upto the last item & say "Whole Square ". [Some say, " All square (D)". It is also ok].

28.3 Activity

28.3.1 a. Teachers let the students write the squares of single digit numbers.

Thus 1^2 = …………
2^2 = …………
3^2 = …………
4^2 = …………
5^2 = ………..
6^2 = ………..
7^2 = ………..
8^2 = ………..
9^2 = ………..
10^2 = …………

Let them do it without using the calculator. Practice how to read also.

b. Teachers, let the students read out the following correctly. Allow the students to check right /wrong among themselves. You can act as a referee.

E.g. $(1 + 2)^2 = 3^2$ = …………The right way of reading this is, "One – plus – two whole – square" " equals " three square "equal to "(nine)"["one – plus – two" should be spoken in one single breath. 'equal' 'equal to ' 'equals', 'is equal to', anything is fine. The word ' whole ' in ' Whole Square ' is important]

28.3.2 Caution: Many Mysore children may not know (no!) ‘ Hole ’ as different from ‘ whole ’. In this book, both these terms occur with important significance. Teachers! Take this opportunity to explain.

28.4 Area and Square:

Finding the squares of numbers is very important. This is because area of a square or a circle needs squares.
A closed figure on a surface has Area.
A Chessboard has 64 squares.
A graph sheet has many big squares and hundreds of small squares.
A Square is a special geometric shape (on a plane surface).Circle, Triangle, Pentagon are also similar geometric shapes and they also have area.[In fact any closed figure on a surface contains area).
Whatever be the shape of the figure, the area contained is expressed in
Square Units
[Sq is used here to mean ‘ square’]
Sq. cm, Sq. m, Sq. feet, Sq. foot, Sq. miles, sq. kms are some of the common terms used in daily life.

28.4.1 Activity
[Teachers! elicit info. from students experiences, the concept of area & its units]
a. Carpenter – for a table/plank.
b. (House) painter – for painting a wall
c. (House) seller – site area
d. From a book) – forest cover in the world
e. (From a book) – area of Karnataka.
f. Many more.

28.5 Calculation (Approximation)
Finding squares of large numbers is not easy.
Actual method is either by using calculator or long multiplication method. But one should always be able to approximate or guess.
Here are some tips (= clues, suggestions) on how to do approximation.(= finding near value)

28.5.1 Worked examples
a. Create your own square grid

$1^2 = 1$ $2^2 = 4$. $10^2 = 100$

Using these, how to approximate?

b. 11^2 =?
$11^2 = 11 \times 11$ this is $> 11 \times 10$
Therefore $11^2 > 110$. Actually 121

c. 20^2 =? $2^2 = 4$ $10^2 = 100$
$(20)^2 = (2 X 10) = 4 X 100 = 400$

d. What is 19^2
$19^2 < 19 \times 20$
i.e. < 380 Actually 361

One can guess, how much less also
192 = 19 x 19
19 x 20 = 380
19 x 19 = 380 – (1 x 19) = 361

e. What is 32^2
$32^2 = 32 \times 32$

$= 4^2 \times 8^2 = 16 \times 64$

Or $(32)^2 > (30)^2$
$> (30)^2 + (2)^2$
> 904 Actually it is 1024

28.5.2 Activity (Game)
One group has calculator. The other goes for mental maths. Use only 2 digit numbers for squaring. [Even upto 20% difference is ok].

28.6 Decimal

28.6.1 Finding squares of decimals.
Teachers, let the students do some simple sums by both the methods i.e.
By multiplying 2 fractions
Or by multiplying 2 decimals (fraction)

a. $(\frac{3}{2})^2$ or $(.5)^2$

$(\frac{3}{2})^2 = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2.25$

$(1.5)^2 = 1.5 \times 1.5 = 2.25$

b. $(\frac{1}{4})^2$ or $(0.25)^2$

$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = 0.0625$

$(.25)^2 = .25 \times .25$

$= \frac{25X25}{10000} = \frac{625}{10000} = 0.0625$

c. Let students select their own, do and show.

28.6.2 Teachers should show that squares of numbers > 1tend to be big.

Square of numbers < 1 (i.e., fractions) tend to be small (less than the original fraction).

Thus 2 x 2 = 4 but 3 x 3 jumps to 9

Similarly $\frac{1}{2} \times \frac{1}{2} << \frac{1}{2}$ (much less than)

$\frac{1}{4} \times \frac{1}{4} << \frac{1}{4}$ (much less than)

28.6.3 Exercises:
Do by all 3 methods:
a. Approximation
b. Actual (manual) rigorous calculation
c. Using calculating machine.

a. 15^2 b. $(1.5)^2$ c. $(0.15)^2$ d. $(15.5)^2$ e. $(21)^2$

f. $(2.1)^2$ g. $(4.2)^2$ h. $(\frac{3}{8})^2$ i. $(2\frac{3}{8})^2$ j. $(\frac{4}{7})^2$

28.7 Square Root:
What is Square root?
Example:
5 x 5 = 25

i.e., $5^2 = 25$

In words, square of 5 is 25.
Square root of 25 is 5. (Reverse, or converse or opposite or ulta, by definition)

28.7.1 **How to write?**

Take a number, n. To denote (= indicate, identify, point out, say) square root of this number, write:

$\sqrt{\mathbf{n}}$ Thus $\sqrt{1}$, $\sqrt{10}$, $\sqrt{100}$, etc.,

[Some persons shorten these in to $\surd n$, $\surd 1$ etc i.e., without the ceiling (= top line, *sir – rekha*) avoid this practice. Not good]

Some people write as $\sqrt[2]{n}$, $\sqrt[2]{100}$ etc. This is really correct and accurate. But not necessary. Only when other roots (eg, cube root) are used we need numbers there.
When big quantities are there, use a very long top-line or use brackets or both.
Eg: $\sqrt{10+15}$ or $\sqrt{(10+15)}$, $\sqrt{(a+b)^2}$, $\sqrt{a^2+b^2}$, or $\sqrt{(a^2+b^2)}$ etc.

28.7.2 How to read?

$\sqrt{n}$. Read it as ' Square root of n' or ' root of n' or simply ' root n'.
If bigger items are written under sq. root:

Eg: $\sqrt{(a^2+b^2)}$ or $\sqrt{a^2+b^2}$
Read it as square root of…$a^2 + b^2$
i.e., give a pause (=time interval, stop and start) after 'root of'. Say the other In one breath (one breath = no stopping, speak continuously) If you say square root of a^2. …Plus b^2, the listener will write as $\sqrt{a}^2 + b^2$.

Exercises: ACTIVITY

A. Let the students make 2 groups. One will read from a list of items. One or two or 3 persons) from the other team will write (as they hear). After 5 items they compare.

B. In A above, reading team becomes writing team & vice versa (= *ulta*)
Sample list:

1. $\sqrt{x}$
2. $\sqrt{(x+y)}$
3. $\sqrt{x} + \sqrt{y}$
4. $\sqrt{x} + y$
5. $y + \sqrt{x}$
6. $\sqrt{x^2+y^2+a^2}$
7. $\sqrt{x}^2 + \sqrt{y^2+a^2}$
8. $x + \sqrt{x^2+y^2+a} + a$
9. $\sqrt{x} + 25$
10. $\sqrt{(x+25)}$

You can make your own.

28.8 **Is square root important?**

Students know the answer to this question. Of course, yes. ['Of course' = Surely' certainly, no doubt, without doubt etc]
Ask a history teacher, " Is history important?"
Ask a engish teacher, " Is grammar important? "
Ask a sot (= drunkard) " Is drinking important? "
They will all say, "Of course, yes".

28.8.1 Square root comes in whenever and whereever areas are used. The idea (= concept) of area in almost all branches of science and certainly in engineering. Even in business or economics squares & square roots occur. There they call it ' doubling'.

28.8.2 Exercises:

In section 28.3.1 we made list of squares of numbers from 1 to 10. Using this let us do some problems.

Worked examples:

1. $\sqrt{100}$ =? See the list Ans = 10.

2. $\sqrt{9X9}$ **=**? One can go to $\sqrt{81}$ = 9 (refer to the list). This is not necessary. By definition, $\sqrt{9X9}$ **=** $\sqrt{9^2}$ = 9.

3. $\sqrt{3X27}$ =? One method is $\sqrt{3X27}$ = $\sqrt{81}$ = 9.

Another method: $\sqrt{3X27} = \sqrt{3X3X9}$ = $\sqrt{9X9}$ = 9.

28.9.1 Examples:

1. $\sqrt{121}$ =? 121 = 11 x 11 = 11^2

$\sqrt{121}$ = $\sqrt{(11)^2}$ = 11 Ans

2. $\sqrt{4x16}$ =? 4 = 2 x 2 = 2^2

16 = 4 x 4 = 4^2

$\therefore$ $\sqrt{4x16}$ = $\sqrt{2^2 \text{ x } 4^2}$ = 2 x 4 = 8 Ans

Exercises:

a. $\sqrt{100}$ **=**?
b. $\sqrt{10X10X10X10}$ **=**?
c. $\sqrt{10000}$ **=**?
d. $\sqrt{5X5X25}$ **=**?
e. $\sqrt{9X25}$ **=**?
f. $\sqrt{3X3X16X25}$ **=**?

28.9.2 Worked examples:

1. $\sqrt{121x^2}$ **=?** 121 = $(11)^2$ $x^2 = (x)^2$

$\therefore$ $\sqrt{121x^2} = \sqrt{(11)^2 \text{x} (x)^2}$ = 11 x x = 11 x

2. $\sqrt{2}$ $\sqrt{2}$ $\sqrt{4}$ $\sqrt{4}$ =?

Ans: By definition $\sqrt{2X2}$ = 2 $\sqrt{4X4}$ = 4

It can be written 4 = $\sqrt{4X4}$ = $\sqrt{(2)^2}$ x $\sqrt{(2)^2}$

Extending this logic 2 = $\sqrt{\sqrt{2} \text{ x } \sqrt{2}}$ $\therefore$ $\sqrt{2}$ x $\sqrt{2}$ = 2

Thus, LHS = 2 X 4 = 8

3. $\sqrt{125}$ =? 125 = 5 x 5 x 5 = 5^2 x 5

$\sqrt{125} = \sqrt{5^2 \text{x} 5}$ = $\sqrt{5^2}$ x $\sqrt{5}$ = 5 x $\sqrt{5}$ Ans.

Exercises:

a. $\sqrt{100a^2}$ **=?**

b. $\sqrt{a^2}\ \sqrt{100}$ **=?**

c. $\sqrt{10000a^2}$ **=?**

d. $\sqrt{5a \text{ X } 5a \text{ X } 25a^2}$ **=?**

e. $\sqrt{a}$ **x** $\sqrt{a}$ **= ?**

f. $\sqrt{a}$ **x** $\sqrt{a}$ **x** $\sqrt{5}$ **x** $\sqrt{5}$ **= ?**

g. $\sqrt{9a^2}$ **x** $\sqrt{25a^2}$ **=?**

h. $\sqrt{3}\ \sqrt{3}\ \sqrt{a}\ \sqrt{a}\ \sqrt{25a^2}$ **=?**

28.10 Approximation method:

Using maggi (= multiplication tables) students can generate a table (= list) of squares.[As in section 28.3.1]. Use this for approximation.

Finding the square root by guessing is ok.

a. $\sqrt{5}$ = ? $2^2 = 4$ $3^2 = 9$

$\therefore$ $\sqrt{5}$ is > 2 and < 3 and it is nearer to 2

$\therefore$ $\sqrt{5}$ is between 2 & 2.5 $\sqrt{5}$ =2.24 (calculator)

b. $\sqrt{11}$ =? $3^2 = 9$; $4^2 = 16$

$\sqrt{11}$ is between 3 and 4.It is nearer to 3. May be 3.2

Let the students check. $\sqrt{11}$ = 3.32 (calculator)

c. $\sqrt{45}$ = ? $6^2 = 36$ $7^2 = 49$

$\sqrt{45}$ is between 6 & 7 and nearer to 7, >6.5 may be 6.8 check.

$\sqrt{45}$ 6.71 (calculator)

d. $\sqrt{3625}$ Approx = 3600

$\sqrt{3625} > \sqrt{3600}$

$> \sqrt{36}\ \sqrt{100}$

> 6 x 10 >60 Very slightly more than 60 may be 60.01(!)

$\sqrt{3625}$ = 60.2 (calculator)

e. $\sqrt{12345} = x$ x =?

$x > \sqrt{12300}$ $\sqrt{121}$ **<** $\sqrt{123}$ **<** $\sqrt{144}$

> $\sqrt{123}\ \sqrt{100}$ $11 < \sqrt{123} < \sqrt{144}$

$> 10\ \sqrt{123}$ May be 11.1

> 10 x 11.1 $\sqrt{123}$ = 11.09 (calculator)

>111 Check. $\sqrt{12345}$ = 111.1 (calculator)

28.11 How to find square root?

28.11.1 Answer to the question above. “Give me a calculator” ok.
Approximation method given in 28.10 above is special to this book. It may not be found in standard books. Since this writer believes in ‘estimations’, it is included here.

28.11.2 Usual methods are:
A. Factorization method
B. Traditional ‘ division method’.

28.12 Factorisation method.
The principle here is to write a given complex number in the form of squares.
Example: $\sqrt{196}$ =? 196 = 4 x 49. Here we have factorised 196 into two factors.
Each one is a square.
$\therefore \sqrt{196} = \sqrt{4X49} = \sqrt{2^2 \times 7^2}$ = 2 x 7 = 14 Ans.

28.12.1 Factorisation method is important and should demonstrated.

a. $\sqrt{14400} = x$

$\therefore$ 14400
= 100 x 2 x 2 x 2 x 2 x 2 x3 x 3
= 10^2 x 2^2 x 2^2 x 3^2
= $\sqrt{14400}$ = 10 x 2 x 2 x 3 = 120

100 | 14400
2 | 144
2 | 72
2 | 36
2 | 18
3 | 9
3

b. $\sqrt{1440} = y$

$y = \sqrt{1440}$
$= \sqrt{10 X 2 X 2 X 2 X 2 X 3 X 3}$
$= \sqrt{10 \times 2^2 \times 2^2 \times 3^2}$
$= \sqrt{10} \times \sqrt{2}^2 \times \sqrt{2}^2 \times \sqrt{3}^2$
$= \sqrt{10}$ x 2 x 2 x 3
= 12 x $\sqrt{10}$ = 12$\sqrt{10}$

c. Let students select their own problems & try to solve by factorizing.

28.12.2 **Exercises:**
Factorise and then find sq. root
Example: 1. $\sqrt{12100}$ **=**?

12100 = 121 x 100
= 11 x 11 x10 x 10
= 11^2 x 10^2

$\therefore$ $\sqrt{12100} = \sqrt{11^2 \times 10^2}$ = 11 x 1 = 110 Ans
$\sqrt{1210} = \sqrt{11^2 \times 10}$ =11 x $\sqrt{10}$ Ans

a. 86436	b. 86436 x^2	c. 36	d. $36a^2b^2$
e. 324	f. 324 x $(a + b)^2$	g. 900	h. 900 x a^2 x $(x - y)^2$
i. 676	j. $\frac{676}{a^2}$	k. 3380	l. 2700
m. 162	n. 18	o. 1764	p. 3380a

q. $18a^2b$ r. $\frac{18}{a^2}$ s. $\frac{1764 \times a^2}{b^2}$ t. $\frac{3380 \times a}{b^2}$

28.13 Rules, Tips

28.13.1 Teachers, drill the following:

a. $x \times x = x^2$

b. $x \times x \times y \times y = x^2 \times y^2$

c. $xy \times xy = (xy)^2 = x^2 y^2$

d. $(xyz)^2 = x^2 y^2 z^2$

e. $(\text{Number} \times a)^2 = (\text{Number})^2 \times a^2$

To self – study student: ‘drill’ means read and write (= do) many times so that you are familiar with all the rules & equations.

28.13.2 Drill:

a. $\sqrt{x} \times \sqrt{x} = x$

b. $\sqrt{x}^2 \times \sqrt{x}^2 = x^2$

c. $\sqrt{(anything)} \times \sqrt{(Something)} = anything$

d. $\sqrt{1025} \times \sqrt{1025} = 1025$

28.13.3 There are formulas for $(x+y)^2$ and $(\text{Number} + a)^2$. Those can be learn’t later. At present

$\sqrt{(x+y)^2} = (x+y)$

$\sqrt{(number + a)^2} = (number + a)$

28.14 Exercises: Say √ / X

a. $\sqrt{5} \times \sqrt{3} = \sqrt{15}$

b. $\sqrt{5^2 \times 3^2} = 225$

c. $\sqrt{5} \times \sqrt{3} = \sqrt{8}$

d. $\sqrt{5^2 \times 3^2} = 15$

e. $\sqrt{5} + \sqrt{3} = \sqrt{8}$

f. $\sqrt{5^2 \times 3} = 75$

g. $\sqrt{5+3} = \sqrt{8}$

h. $\sqrt{5^2 \times 3} = 15$

i. $\sqrt{5+3} = 2\sqrt{2}$

j. $\sqrt{5^2 \times 3} = 5\sqrt{3}$

28.15 Traditional method of finding square root – OR – “Square Division method” or “Double Division Method”

28.15.1 Exactly finding square root by traditional method is Not Necessary in this manual. If teachers find some bright students asking for it, here are some:

a.

	3.4	6
3	12.00	00
	9	
64	300	
	256	
686	4400	
	4116	

$\sqrt{12} = 3.464$ (calculator)

b.

	32
3	1024
	9
62	124
	124
	0

$\sqrt{1024} = 32$

c.

	25
2	625
	4
45	225
	225
	0

$\sqrt{625}$ =25

Exercises: [Only for advanced students] Do by the above method.

i. $\sqrt{225}$ ii. $\sqrt{324}$ iii. $\sqrt{676}$ iv. $\sqrt{86436}$

v. $\sqrt{14400}$ vi. $\sqrt{1440}$ vii. $\sqrt{2700}$

1. Square root of 5

	2.234
2	5.00 00 00
	4
42	100
	84
443	1600
	1329
4464	27100
	≈17856

$\sqrt{5} \approx$ 2.23

2. $\sqrt{11}$

	3.31
3	11.00 00
	9
63	200
	189
661	1100
	661
	439

$\sqrt{11}$ > 3.31

3. $\sqrt{45}$

	6.708
6	45.00 00 00
	36
127	900
	889
1308	1100 00
	107264

$\sqrt{45} \approx$ 6.708
= 6.71

Can also be: $\sqrt{45} = \sqrt{9 X 5} = 3\sqrt{5}$
$\approx$ 3 X 2.234 $\approx$ 6.702

4. $\sqrt{12345}$ =?

	111.1
1	1,23,45.00
	1
21	23
	21
221	245
	221
2221	2400
	2221

$\sqrt{12345} \approx$ 111.1

5. $\sqrt{10} = ?$

```
        3.162
   3 | 10.00 00 00
     |  9
  61 | 100
     | 61
 626 |  3900
     |  3756
 632 | 14400
     | 12644
     |
```

$\sqrt{10} \approx 3.162$

6. $\sqrt{1440} = ?$

```
       38
   3 | 1440
     | 9
  68 |  540
     |  544
     | ≈ 38
```

$\sqrt{1440} = \approx 38$

Can also be done as: $\sqrt{1440} = \sqrt{144} \times \sqrt{10} = 12 \times \sqrt{10}$
$= 12 \times 3.162 = 37.94$

===

Chapter - 29 Formulas of Algebra

29.1 Note to the teacher: In this manual, 3 formulas of algebra are selected out as very important. They are:
1. $(a + b)^2$ 2. $(a – b)^2$ 3. $(a + b)(a – b)$
Of these 1 and 2 are combined as $(a \pm b)^2$. Please help the student memorize properly. There may be other formulas, which may be equally important. Such as $(a + b)^3$, $(a + b + c)^2$, $a^3 + b^3$ etc. Ours is a very simple introduction to mathematics.

29.2 Formulas:

$(x + y)^2$ and $(x - y)^2$

These two formulas are so important and so useful in many estimations, it is useful to memorize.
Let the students memorize
$(x + y)^2 = x^2 + 2xy + y^2$
$(x - y)^2 = x^2 - 2xy + y^2$

Now let them ignore these two and memorize only one:

$$(x \pm y)^2 = x^2 \pm 2xy + y^2$$

29.2.1 Exercise:

$(a \pm b)^2 =$ $(p \pm q)^2 =$ $(u \pm v)^2 =$
$(a + b)^2 =$ $(p + q)^2 =$ $(u + v)^2 =$
$(a - b)^2 =$ $(p - q)^2 =$ $(u - v)^2 =$

29.3 Bracket Removal: Multiplication in algebra is already known to the student. Using this knowledge, how to handle brackets is the subject of this paragraph.
$a \times a = a^2$
$a \times b = ab$
$a \times c = ac$

$a(a + b) = a \times a + a \times b = a^2 + ab$
$a(a + b + c) = a \times a + b \times a + a \times c = a^2 + ab + ac$
Now, give many examples, some purely on algebra, some purely with numbers, some both.

29.3.1 Examples:

1. Let us verify a (a+b) = a2 + ab
   Let a=3, b=5
   LHS = 3(3+5) = 3 x 8 = 24

   RHS = $(3)^2$ + (3) x (5)
   = 9 + 15
   = 24
   = LHS Verified

2. Let us verify a (a + b + c) = a2 + ab + ac
   Let a = 3, b = 5, c = 7
   LHS = 3(3+5+7)
   = 3 x 15
   = 45
   RHS = $(3)^2$ + (3) (5) + (3) (7)
   = 9 + 15 + 21
   = 45
   LHS = RHS (verified)

29.3.1 Exercises: Expand or remove brackets.

x (a + b); x (a – b); x $(a^2 – c)$

4 (5 + 10); 4(5342 – 1234); $4(5^2 - 20)$

$2x$ $(y + 3)$; $4ax$ (b + 2); $ax^2(y - 4)$

29.4 Double Brackets: Let us take an example: $(x + a) \times (x + b)$

$(x+a)(x+b)$
$= x\,(x + b) + a\,(x + b)$
$= x^2 + bx + ax + ab$

How to do $(x + a)\,(x + a)$
$= x\,(x + a) + a(x + a)$
$= x^2 + ax + ax + a^2$
$= x^2 + 2xa + a^2$

Exercises: Expand

1. $(x + 4)$ (5 + 10)
2. $(x + 4)$ (5342 – 1234)
3. $(2x + 1)\,(y + 3)$
4. $(4ax + 1)$ (b + 2)
5. $(x + 1)\,(a^2 – c)$
6. (4 – 3) $(5^2 – 20)$
7. $(ax^2 + 4)\,(y - 4)$

29.5 We have seen that $(x + b)\,(x + b) = x^2 + 2x + a^2$
Now make $a = y$
$(x + y)\,(x + y)$
$= x^2 + 2xy + y^2$
Now make $x = a$ $y = b$
$(a + b)\,(a + b) = a^2 + 2ab + b^2$

Exercises:

1. $(x - a)\,(x - b)$
2. $(x - a)\,(x - a)$
3. $(x - y)\,(x - y)$
4. $(a – b)^2$
5. (5 – a) (5 – b)
6. (5 – a) (5 – a)
7. (5 – y) (5 – y)
8. (5 – 3) (5 – 3)
9. $(6 – b)^2$

10. Your own questions.

29.6 2 Important Formulas:
$(a \pm b)^2 = a^2 \pm 2ab + b^2$
$(a + b)(a - b) = a^2 - b^2$

29.7 Using $(a \pm b)^2$

Worked Examples:
1. We know $(13)^2 = 169$. Now let us do it using the formula

$(13)^2 = (10 + 3)^2$
$= (10)^2 + 2(10)(3) + (3)^2$
$= 100 + 60 + 9$
$= 169$

2. $(499)^2 = (500 - 1)^2$
$= (500)^2 - 2(500)(1) + (1)^2$
$= 250000 - 1000 + 1$
$= 259001$

Exercises:

1. $(99)^2$ 2. $(101)^2$ 3. $(49)^2$ 4. $(51)^2$
5. $(1002)^2$ 6. $(505)^2$ 7. $(22)^2$ 8. $(18)^2$
9. $(112)^2$ 10. $(88)^2$

29.8 $(a^2 - b^2) = (a + b)(a - b)$ Formula

Another useful formula is $(a + b)(a - b) = a^2 - b^2$
Same method as before
LHS = a (a – b) + b (a – b)
= ………
= ……..
$= a^2 - b^2$
Students can fill up

29.8.1 Example: Find 501 x 499
501 x 499 = (500 + 1) (500 – 1) Apply Formula
$= (500)^2 - (1)^2$
Here a = 500 b = 1
∴ LHS = 250000 – 1
= 249999

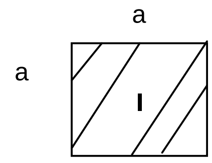
Exercises: Find the values using shortcut formulas

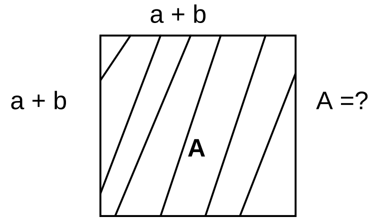
a. 99 x 101 b. 95 x 105 c. 92 x 108 d. 42 x 58
e. 48 x 52 f. 10005 x 9995

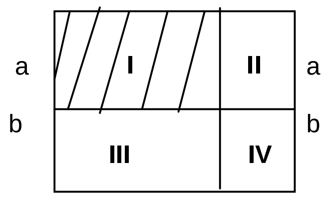
29.9 Some ‘Practical Proofs’

29.9.1 Geometrical ‘Proof’ $(a + b)^2$
a2 = square = area of a square with side a
$(a + b)^2$ = square = area of a square with side (a + b)

Draw these



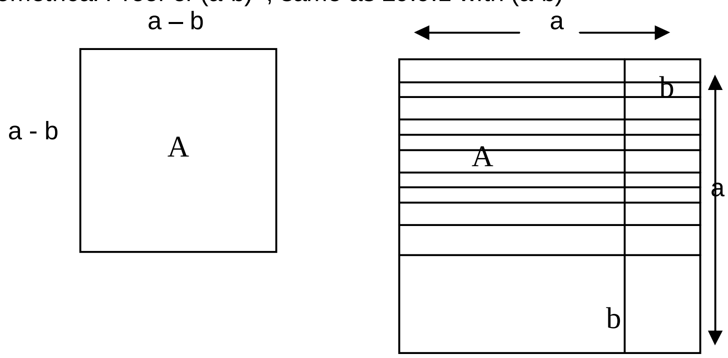




I = a^2
II = Area of rectangle = ab
III = Area of rectangle = ab
IV = Area of square = b^2

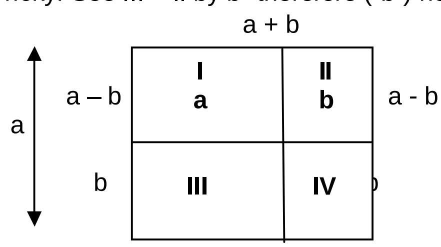
Therefore A = (a + b) 2
= I + II + III + IV
= a^2 + ab + ab + b^2
= a^2 + 2ab + b^2

29.9.2 Geometrical Proof of $(a-b)^2$; same as 29.9.1 with $(a-b)^2$

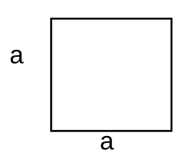


Show that A = I.
Here one has to cut and show.
For this purpose, take two identical pieces (tricky! be careful – b^2 is taken away twice, therefore should put back once.

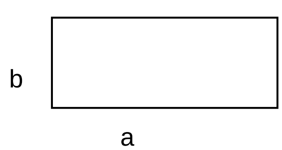
29.9.3 Geometrical Proof of (a + b) (a – b). Do as in the two previous (=earlier, above cases). (Tricky! See III > II by b^2 therefore ($-b^2$) needed)



29.9.4 For those students who have not yet understood these geometries:

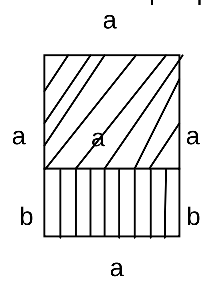


This shape is a square. Its side is a. Its area = a^2
Its area = a^2
= (a x a)

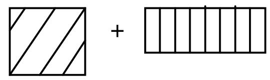


This shape is a rectangle. Its sides are ‘a’ & ‘b’
Its area = a x b

Now see 2 shapes put together (=joined, attached)



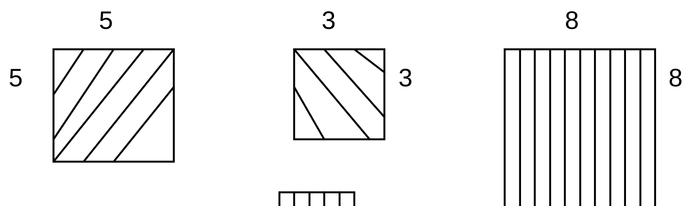
What is the total area?

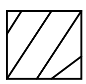
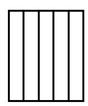


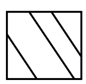
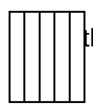
$= a^2 + ab$

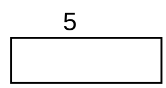
29.10 Activity
All the ideas given in 29.9 can be done by house or classroom activity. To prove $(a + b)^2$ formula. Let a = 5 cm b = 3 cm

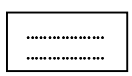
Draw a square of 8 cm (i.e., a + b) on a cardboard. Make 3 pieces (cut neatly).



Place  inside  the big one.

Place  also inside  the big one.

There is still empty space left. Now make 3 

2 pieces. Now if you put  into the big piece, one after the other, you will see it will tightly fit.

29.11 Activity
Cut and paste method of 29.10 can be directly done on a graph sheet. Do the same steps as in 29.10. Find the gaps (& their areas) by counting the small squares (of the graph sheet).

29.12 Activity
Earlier sections showed $(a + b)^{2.}$ Similarly practical constructions can be made for $(a – b)^2$ also. With a little difficulty (a + b) (a – b) also.